4. S. Ya. Yarema, "The methodology of determining the characteristics of resistance to crack development in materials subjected to cyclical loads," Fiz. Khim. Mekh. Mater., No. 4 (1981).
5. E. Ionnidics and T. Harris, "A new model of fatigue life in rocker bearings," Problemy Treniya i Smazki, 107, No. 3 (1985).
6. T. Tanaka, T. Sakai, and K. Okada, "A statistical study of fatigue-life distribution based on the coalescence of cracks from surface defects," Trans. ASME, Ser. A, 50, No. 454 (1984).
7. M. Ichikawa, "Some problems in probabilistic fracture mechanisms," Trans. ASME, Ser. A, 50, No. 456 (1984).
8. H. Kashimura and N. Tsushima, "Improvement of rolling contact fatigue life of bearing steels," Techn. Pap. Ser./SAE; No. 841123, p. 1 (1984).
9. P. Bowen, S. G. Druce, and J. F. Knott, "Effects of microstructure on cleavage fracture in pressure vessel steel," Acta Metallurgica, 34, No. 6 (1986).
10. J. M. Rodriguez Ibabe and J. Gil Sevillano, "Fatigue crack path in medium-high carbon ferrite-pearlite structures," in: Advanced Fracture Research, Proc. 6th Int. Conf. Fracture (ICF6), New Delhi, 1984, Oxford (1984), No. 3.
11. M. Schaper and D. Bösel, "Rasterelektronenmikroskopische in-situ-Untersuchungen der Ermüdungstrissausbreitung in metallischen Werkstoffen," Prakt. Metallographie, 22, No. 4 (1985).
12. V. Z. Parton and E. M. Morozov, The Mechanics of Elastic-Plastic Destruction [in Russian], Nauka, Moscow (1985).
13. Foscamp, "Changes in a material subjected to contact loads during pumping," Problemy Treniya i Smazki, 107, No. 3 (1985).
14. H. Nakashima, K. Maeda, N. Tsushima, and H. Muro, "Application of x-ray measurement to bearing failure analysis," Techn. Pap. Ser./SAE: No. 830825 (1983), p. 1.
15. V. V. Bolotin, Statistical Methods in Structural Mechanics [in Russian], Stroiizdat, Moscow (1965).
16. V. V. Bolotin, "An energy approach to the description of fatigue crack growth in the nonuniaxial stressed state," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1985).
17. V. M. Finkel', Physical Bases for Retarding Destruction [in Russian], Metallurgiya, Moscow (1977).

AN ULTRASOUND METHOD FOR EXPERIMENTAL EVALUATION OF FIELD NONUNIFORMITIES IN INTERNAL DYNAMIC STRESSES

V. P. Lebedev and V. M. Rodyushkin UDC 534.1

The dynamically stressed state of machine elements and structures is determined through the measurement of the vibrations at the surfaces of these elements. Data relating to the structure of the elastic field within these elements are obtained through sequential calculations [1] based on mathematical relationships known to us from the theory of elasticity. These methods are based on measurements and calculations which have proved themselves in evaluating the structure of a static and quasistatic elastic field, but they become virtually useless when consideration must be given to the wavelike nature of the field. However, an increasing number of problems is encountered in engineering, where it is precisely these wave processes in machines and constructions that must be subjected to study [2]. There arises a need to find new principles for the experimental evaluation of field structure.

1. Let us turn to the studies [3, 4] where it is proposed to use the phenomenon of nonlinear interaction between elastic waves. The essence of this proposal lies in the fact that a plane monochromatic ultrasonic wave, on reaching a zone of a rather powerful

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field, is modulated with respect to phase at such a time. The effect of modulation is accumulated over the extent of the entire wave path through the zone and depends on the direction of wave propagation. Thus, we obtain information regarding the structure of a low-frequency elastic wave field within an element of the construction. The general form of the mathematical connection between the instantaneous increment in the phase of the acoustic signal with the dynamic deformations to which this field is subjected in the case of an isotropic solid body has been derived in [3] under the condition that the time scale for the change in the dynamic deformations is considerably larger than the period of the ultrasonic wave, while the displacement amplitude of this field is represented by the displacements generated by the ultrasonic wave:

$$\varphi(t) = B\int_0^{z_0'} (\beta\varepsilon_{11}' + \gamma\varepsilon_{22}' + \gamma\varepsilon_{33}')\big|_{t=t_0+z'/c}\, dz', \tag{1}$$

where $B = \omega/(2\rho c^2)$; ω is the frequency of the ultrasound wave; ρ is the density; c is the speed of ultrasound propagation; $\beta = 6\mu + 3\lambda + 4m + 2\ell$, $\gamma = \lambda + 2\ell$ are the parameters of material nonlinearity; μ, λ, ℓ, m, and n are the 2nd and 3rd order elasticity constants, respectively, ε_{ii}' represents the diagonal components of the strain tensor for the elastic dynamic field.

In the following we examine a method for the evaluation of field nonuniformities in the internal dynamic stresses within machine and structural elements, said method based on utilization of relationship (1), suitable for practical applications.

We note that formula (1) has been derived in such a rather simple form because of our choice for the coordinate system: the z' axis is directed along the path of wave-front propagation for the ultrasound probing wave. Since the object of our study is the field within the structural element, it will be expedient in the following to examine condition (1), expressed in a system of coordinates connected to that element. It is sufficient to write the components ε_{ij}' in this new system of coordinates and correctly to carry out the substitution of the integration variable. Then

$$\varphi(\Theta, t) = B\sum_{i=1}^{3}\sum_{j=1}^{3} u_{ij}(\Theta)\int_0^{z_0} \varepsilon_{ij}\left(x(z), z, t + \frac{z}{c}\right)\frac{1}{\cos\Theta}\, dz, \tag{2}$$
$$u_{ij}(\Theta) = \beta A_{1i}A_{1j} + \gamma A_{2i}A_{2j} + \gamma A_{3i}A_{3j}$$

[$A_{ij} = A_{ij}(\Theta)$ is the turning matrix of the coordinate system (see Fig. 1)].

Relationship (2) serves as the initial equation for the development of methods to evaluate the structure of the dynamic elastic field. Here the advance of the phase $\varphi(\Theta, t)$ serves as the integral characteristic of the field of strains ε_{ij}. The problems dealing with evaluation of the structure of the differential characteristic on the basis of known integral characteristics of the process (of the field) as a rule, are not well founded. The solution will be sought in a narrow class of functions so as to ensure, on the one hand, reliable differences in the unknown structural features of the field and, on the other hand, a probable physical interpretation of these features.

In our case, in connection with the wave elements of machinery, constructions of such a class of functions (the mathematical model of the field) are given in the form

$$\varepsilon_{11}(x, z, t) = \varepsilon_0(z)\exp[i(\Omega t - kx)],$$
$$\varepsilon_{22} = \varepsilon_{33} = -\nu\varepsilon_{11},\ \varepsilon_{ij} = 0 \text{ when } i \neq j \tag{3}$$

(ν is the Poisson coefficient, and Ω and k are the frequency and wave number of the wave, being considered here, as it propagates through the element). This model of the field, or the quasi-rod approximation, represent a traveling wave that is uniform along y and nonuniformly distributed along z (the transverse coordinate of a rod of finite thickness). We will take the correlation interval τ of the function $\varepsilon_0(z)$ as the parameter of nonuniformity, characterizing the unique features of the field structure. Through substitution of (3) into (2) we obtain

$$\varphi(\Theta, t) = B(\Theta) \exp[i\Omega t_0] \int_0^{z_0} \varepsilon_0(z) \exp\left[iz\left(\frac{\Omega}{c\cos\Theta} - k\,\mathrm{tg}\,\Theta\right)\right] dz,$$

$$B(\Theta) = \frac{u_{11}(\Theta) - \nu u_{22}(\Theta) - \nu u_{33}(\Theta)}{\cos\Theta} B.$$

We are interested in the energy characteristic $\varphi(\Theta, t)$, since it reflec s the energy relationships of the field, and its energy structure. Moreover, when the field being studied is monochromatic, it is the square of the modulation index, i.e., a quantity used in engineering and both familiar and accessible to measurement:

$$m_\varphi^2 = \varphi(\Theta, t)\overline{\varphi(\Theta, t)} = B^2(\Theta) \int_0^{z_0}\int_0^{z_0} \varepsilon_0(z)\bar{\varepsilon}_0(z_1) \exp[iU(z - z_1)]\, dz\, dz_1,$$

$$m_\varphi^2 = B^2(\Theta) \int_0^{z_0} K(\Delta) \exp[iU\Delta]\, d\Delta = B^2(\Theta) G(U). \tag{4}$$

Here $\Delta = z - z_1$; $U = \frac{\Omega}{c\cos\Theta} - k\,\mathrm{tg}\,\Theta$;

$$G(U) = \int_0^{z_0} K(\Delta) \exp[iU\Delta]\, d\Delta; \tag{5}$$

$$K(\Delta) = \int_{-\Delta}^{z_0 - \Delta} \varepsilon_0(z + \Delta)\bar{\varepsilon}_0(z_0)\, dz. \tag{6}$$

When we take into consideration that $\varepsilon_0(z) = 0$ when $z < 0$ and $z > z_0$, in (5) and (6) we can set the integration limits from $-\infty$ to $+\infty$, i.e., $K(\Delta)$ and $G(U)$ are the correlation function and power spectrum of the function $\varepsilon_0(z)$ (U is the spatial frequency). Since $\varepsilon_0(z)$ describes the nonuniformity of the field at a section in the element, $K(\Delta)$ and $G(U)$ are equally likely and adequate characteristics of this nonuniformity. It follows from relationships (4) and (5) that based on the angular relationship m_φ^2, accessible to measurement, we can find $G(U)$ and $K(\Delta)$, and consequently we can also find τ. We determine the correlation interval in various ways, primarily through consideration of the specific nature of the signals being processed, as well as from considerations of computational convenience. In analytical studies one of the following relationships is taken as τ:

$$\tau = \left(\int_{-\infty}^{\infty} \Delta^2 K^2(\Delta)\, d\Delta \Big/ \int_{-\infty}^{\infty} K^2(\Delta)\, d\Delta\right)^{1/2}, \quad \tau = \int_{-\infty}^{\infty} |K(\Delta)|\, d\Delta / K(0).$$

2. The experimental estimate $\varepsilon_0(z)$ of the nonuniformity reduces to the measurement of the modulation index m_φ^2 for various angles of ultrasound-wave propagation through the subject waveguide element of the machine, to the finding of $K(\Delta)$ from (4) and (5), which involves utilization of the Fourier transformations and the calculation of the nonuniformity parameter τ.

The measurements of the modulation index were carried out in accordance with the diagram shown in Fig. 1, where 1 represents the generator of monochromatic ultrasonic oscillations providing for frequency stability no lower than 10^{-7}; 2 identifies the converter prisms, 3 represents the piezoceramic plate with a resonant frequency of 2 MHz; 4 is the model of the waveguide element of the machine; 5 is a band filter; 6 is a selective voltmeter; 7 is the piezoceramic emitter producing the elastic dynamic field; and 8 identifies the low-frequency generator. The probe angle Θ was altered by means of a set of pairs of prisms. The modulation index was defined as the ratio between the amplitude of the side component in the spectrum of the incoming modulated signal to the amplitude of the carrier, measured by means of the selective voltmeter. The error in the measurement of the modulation index in this case amounted to 12% in the experiment. We used an arm made of plastic with dimensions of 70 × 150 × 1500 mm as the model of the waveguide element in which a variable elastic field was excited at one end. Owing to the choice of a sufficiently high frequency of oscillations (from 10 to 50 kHz) a traveling wave appeared within this arm, and this wave was virtually attenuated at a length of 1.5 m and, in our opinion, corresponded fully to the field model in (3). In the process of working with the experimental data we employed a computational method to offset the constant component $\varepsilon_0(z)$. The nonuniformity of the field was evaluated relative to the uniform distributions of dynamic strains through a section of the element, i.e., the ratio τ/τ_0 served as the characteristic, and this characteristic for a uniform distribution $[\varepsilon_0(z) = \text{const}, \tau = \tau_0]$ is equal to unity. Some cumbersome calculations were the price we had to pay for clarity of results.

The quantity τ/τ_0 was determined experimentally as a function of the distance to the emitter, as a function of the frequency of the dynamic strains, as a function of the presence of coupling within the arm, the latter being modeled by glueing a piece of plastic to this arm. Observations yielded the following results. The correlation interval is at a minimum near the emitter: $\tau/\tau_0 = 0.76$ (at a frequency of 20 kHz), which indicates maximum nonuniformity in this region. From the relationship between the values of τ/τ_0 and ℓ, shown in Fig. 2 (the distance between the point of measurement and the emitter, generating the dynamic elastic field), we can see that in the process of transition to regions within the models of the elements more distant from the emitter the structure of the field becomes smoother, nearly uniform in distribution. At a distance $\ell = 5(2\pi)/K$ the correlation interval increases by 20-25%.

Measurements at various frequencies (15, 20, 30, 40, and 55 kHz) showed a reduction in the correlation interval from 0.95 to 0.22. This gives evidence to the effect that with an increase in frequency there is an increase in the nonuniformity of the field (see Fig. 3). At frequencies of 15 kHz and lower, within the limits of error for the method, the structure of the field at various points in no way differs from uniform distribution. We also noted an increase in the nonuniformity of the field near the coupling: in the experiments this reduction in the correlation interval amounted to 50% in comparison with the given period.

The above-enumerated results are in qualitative agreement with presently existing concepts regarding the propagation of oscillations in elastic waveguides [5, 6]. Moreover, with multiple repetitions of the experiment the results are repeated. Scattering in the values of τ/τ_0 for various series of measurements, given identical conditions, does not exceed 15%. This provides a basis for speaking of a quantitative estimate of the magnitude of the nonuniformity on the basis of the parameter τ. We should note that in addition to the nonuniformity of the field in a section of the element on the basis of the cited data, the weak nonuniformity of the field along the element is also evaluated.

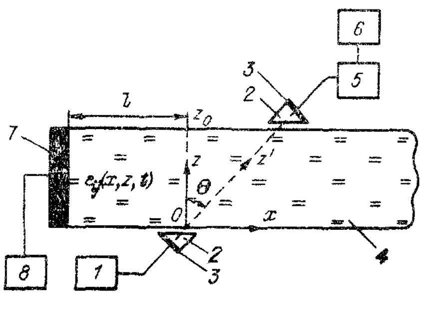

Fig. 1

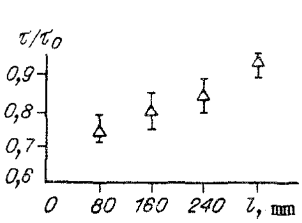

Fig. 2

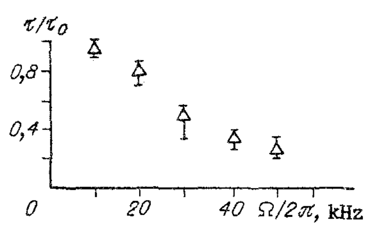

Fig. 3

The proposed method can be used for nondestructive determination of the internal dynamic stressed state of machine components, constructions, and their elements.

In conclusion, let us note that model (3) represents a rough reflection of field nonuniformity, and fails to fully describe the real dynamic stresses within the waveguide. However, it does provide a solution for the poorly founded problem, one that is suitable for practical application. Moreover, this method of evaluating the nonuniformities of the field allows for measurement tools currently in production and, consequently, immediately available in actual practice.

LITERATURE CITED

1. N. I. Prigorovskii, Methods and Means of Determining the Fields of Strains and Stresses: Handbook [in Russian], Mashinostroenie, Moscow (1983).
2. A. I. Vesnitskii and A. I. Potapov, "Certain properties common to wave processes in one-dimensional mechanical systems of variable lengths," Prikl. Mekh., 11, No. 4 (1975).
3. B. A. Konyukhov and G. M. Shalashov, "Nonresonant parametric interactions of elastic waves in an isotropic solid medium," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 5 (1976).
4. O. V. Malyukov and V. M. Rodyushkin, "The use of ultrasound to measure dynamic forces," Mashinovedenie, No. 2 (1988).
5. V. T. Grinchenko and V. V. Meleshko, Harmonic Oscillations and Waves in Elastic Bodies [in Russian], Naukova Dumka, Kiev (1981).
6. V. I. Erofeev, I. D. Konyukhova, and V. M. Rodyushkin, "The propagation of waves in crossed rod systems," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1987).

LONG WAVES OF FINITE AMPLITUDE IN POLYDISPERSED GAS SUSPENSIONS

N. A. Gumerov

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Most theoretical studies dealing with the wave dynamics of gas suspensions are devoted to the propagation of weak waves and waves of finite amplitude in monodisperse mixtures [1-7]. In [1, 8] we find a model for a polydisperse suspension consisting of a gas and a finite number of particle fractions. The generalization of this model to the continuous functions of particle distribution by size insofar as this relates to description of the propagation of sound waves and vapor and gas suspensions, as well as certain of the results from the calculation of dispersion and attenuation of monochromatic perturbations, is presented in [9].

It has been demonstrated in the present study that the propagation of long-wave perturbations of finite amplitude in rarefied polydisperse gas suspensions with an arbitrary mass content of particles within the mixture and an arbitrary function of particle distribution by size can be described within the framework of the model of a monodisperse medium with a particular effective particle radius. In particular, this allows us to generalize the results of the earlier analytical and numerical studies into the propagation of long waves in monodisperse suspensions without phase transitions to polydisperse gase suspensions.

1. Original Equations. Let us examine a rarefied gas suspension with a limited volumetric particle content of $\alpha_2 \ll 1$ within the mixture. The relative mass particle content $m = \alpha_2\rho_2{}^0/\rho_1{}^0$ in this case need not be small, since the true density of the material for the particle is considerably greater than the true gas density $\rho_2{}^0 \gg \rho_1{}^0$. We will assume the particles to be incompressible, and that the gas is ideal, calorically perfect (the viscosity and thermal conductivity of the gas is taken into consideration only in interphase interaction).

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